

Differential neutrino heating and reduced ^4He production from decaying particles in the early universe

Richard E. Leonard and Robert J. Scherrer

Department of Physics, The Ohio State University, Columbus, Ohio 43210

Abstract

A decaying particle that dominates the energy density of the universe and then decays with a lifetime $\sim 10^{-1}$ sec into electromagnetically-interacting particles can heat the background electron neutrinos more than the μ and τ neutrinos, resulting in a decrease in the primordial ^4He abundance. Previous calculations of this effect assumed sudden decoupling of the neutrinos from the electromagnetic plasma. Here we repeat this calculation using an improved (although still approximate) treatment of the energy flow from the blackbody radiation into the various neutrino components. We find that with our improved calculation, the reduction in the primordial ^4He is not as large as had been previously thought, but it occurs over a larger range in decaying particle lifetime τ . The reduction in ^4He can be as large as $\Delta Y = -0.01$ for $\tau = 0.1 - 0.7$ sec.

I. INTRODUCTION

The effects of decaying particles on primordial nucleosynthesis have been investigated by many authors (see, for example, [1] - [2] and references therein). One interesting effect, first noted by Scherrer & Turner [3] can occur if the particle decays into products which interact electromagnetically and go into thermal equilibrium with the cosmic blackbody background. If such decays occur well after the neutrinos have dropped out of thermal equilibrium (at a temperature of a few MeV) then the neutrinos do not share in the heating of the blackbody background. For lifetimes much shorter than the neutrino decoupling temperature, the neutrinos share fully in the heating of the blackbody background. In the intermediate regime (lifetimes on the order of 0.1 sec), an interesting effect can occur. Because the electron neutrino couples more strongly to the e^-e^+ pairs than do the μ and τ neutrinos, it decouples at a slightly lower temperature. Thus, for lifetimes in this intermediate range, it is possible for the decaying particle to heat the electron neutrinos more than the μ and τ neutrinos. This results in a net decrease in the expansion rate relative to the weak ($n \leftrightarrow p$) rates, giving a reduction in the ${}^4\text{He}$ abundance [3]. This effect can occur, for example, in gravitino decay [4].

This possibility of reducing the primordial ${}^4\text{He}$ production is quite interesting, because recent calculations suggest that the standard model of big bang nucleosynthesis can be made consistent with current observations only if the “true” primordial ${}^4\text{He}$ abundance is larger than currently believed, or if primordial nucleosynthesis is modified to give a smaller production of ${}^4\text{He}$ [5]. However, the treatment of this effect in references [3]-[4] was quite crude: the neutrinos were assumed to decouple suddenly when their interaction rates dropped below the expansion rate; entropy dumped into the system before decoupling was shared equally by the neutrinos, photons, and e^-e^+ pairs, while entropy released after decoupling went entirely into the photons and e^-e^+ pairs. Hence, we are motivated to do a more careful investigation, involving a more accurate treatment of the transfer of energy from the decaying particle to the neutrinos. We use the linearized energy-transfer equation given by Rana and Mitra [6], based on earlier work by Herrera and Hacyan [7]. We do not calculate the full distortion in the spectrum, but instead treat each species of neutrino as a black body with a single temperature. Although this is not as sophisticated as the treatment given by Dodelson et al. [2], it represents a significant improvement over other previous studies of this problem. (Dodelson et al. examined only decaying neutrinos, which

do not have a large enough energy density to make this an interesting effect).

In the next section, we present our calculation of the evolution of the neutrino temperature in the presence of a decaying particle with a short ($10^{-2} - 2$ sec) lifetime. In Sec. III, we calculate the effect of the differential neutrino heating on the primordial ^4He abundance, and briefly summarize our conclusions. We find that the reduction in ^4He production is smaller than had been previously thought, but it is still large enough to be interesting.

II. EVOLUTION OF THE NEUTRINO TEMPERATURE

The calculation begins by adding to the standard cosmological model an X particle which decays exclusively into particles that thermalize rapidly compared with the expansion timescale. This particle adds a contribution to the total energy density driving the expansion according to the Friedmann equation

$$H \equiv \frac{\dot{R}}{R} = \left(\frac{8}{3}\pi G\rho\right)^{1/2}. \quad (1)$$

where R is the expansion scale factor. Here the total density $\rho = \rho_{std} + \rho_X$ with ρ_{std} representing the energy density in the standard model. The X density ρ_X evolves according to the equation

$$\dot{\rho}_X = -\Gamma\rho_X - 3H\rho_X, \quad (2)$$

where Γ is the decay rate, $\Gamma \equiv 1/\tau$, and τ is the X particle lifetime. This equation has the solution

$$\rho_X = \rho_{X0} \left(\frac{R}{R_0}\right)^{-3} e^{-t/\tau}. \quad (3)$$

We consider particle lifetimes in the range $10^{-2}\text{sec} < \tau < 2$ sec. Following reference [3], we quantify the particle number density by the parameter r , which gives the number density of the X particles relative to photons before e^-e^+ annihilation. The only quantity which determines the energy density of the X particles as a function of temperature is rm_X , where m_X is the particle mass. In all of our calculations, we take $rm_X = 10^4$ MeV. For this choice of rm_X and particle lifetimes above, the density of the X particles dominates the expansion prior to decay. More importantly, the entropy produced by the decay dominates the previously-existing radiation during nucleosynthesis. When these two conditions are satisfied, the evolution of the element abundances is independent of rm_X , and the final results will be essentially a function only of τ [3]. For the range of lifetimes considered here, these conditions will be

satisfied for $rm_X \gtrsim 10^3$ MeV, so this is the range over which our results will be applicable (See ref. [3]). For comparison, a massive tau neutrino has a maximum rm_X of about 1 MeV (for $m_\nu \sim 3 - 5$ MeV), so we are examining a very different region of parameter space from that discussed in ref. [2].

The decay of the X particle heats the electromagnetic plasma. Let ρ_γ and p_γ be the energy density and pressure of the photons and all relativistic particles in thermal equilibrium with the photons. Then the evolution of ρ_γ is given by

$$\dot{\rho}_\gamma = \Gamma\rho_X - 3H(\rho_\gamma + p_\gamma). \quad (4)$$

In references [3]-[4], the authors used a sudden decoupling approximation to deal with neutrino heating. In this approximation the neutrinos are assumed to have blackbody spectra with temperatures T_{ν_i} , $i = e, \mu, \tau$. The temperature of an individual neutrino species is set equal to the photon temperature until it decouples and thereafter the neutrino temperature decreases as $1/R$. The decoupling condition used in references [3]-[4] is $\Gamma_{\nu_i}/H < 1$ where $\Gamma_{\nu_i} = A_{\nu_i}T^5$ ($i = e, \mu, \tau$). For consistency with our derivations below, we re-express Γ_{ν_i} in the form

$$\Gamma_{\nu_i} = \alpha_{\nu_i}G_F^2T^5, \quad (5)$$

where G_F is the Fermi coupling constant, and α_{ν_i} is a dimensionless constant of order unity. The values for A_{ν_i} used in references [3]-[4] correspond to $\alpha_{\nu_e} = 0.3$ and $\alpha_{\nu_{\mu,\tau}} = 0.07$. This is obviously a very crude approximation and is considered here only for comparison with our new calculations.

In this work we follow the treatment of Rana and Mitra [6] and Herrera and Hacyan [7], who treated the neutrinos as a perfect blackbody and calculated the rate of energy transfer from the electron-positron pairs to the neutrinos through scattering and annihilation. Let u_{ν_i} be the total rate of energy density transfer from the e^-e^+ pairs to a single neutrino species ν_i ($i = e, \mu, \tau$) via annihilations and scatterings. Then equation (4) is modified to

$$\dot{\rho}_\gamma = \Gamma\rho_X - \sum_{i=e,\mu,\tau} u_{\nu_i} - 3H(\rho + p)_\gamma \quad (6)$$

while the neutrino density evolves as

$$\dot{\rho}_{\nu_i} = u_{\nu_i} - 4H\rho_{\nu_i}, \quad (7)$$

In the limit of small temperature difference between the electrons and neutrinos, u_{ν_i} is given by [6]

$$u_{\nu_i} = 2I_{e^- \rightarrow \nu_i}(T_\gamma - T_{\nu_i}) \quad (8)$$

where $I_{e^- \rightarrow \nu_i}$ is a function of T_γ which gives the rate of energy density transfer per unit temperature difference from electrons into a particular neutrino species ν_i , and the factor of 2 comes from including both electrons and positrons. In the limit of large temperatures ($T_\gamma \gg m_e$), I has the form

$$I_{e^- \rightarrow \nu_i} = \beta_{e^- \rightarrow \nu_i} G_F^2 T_\gamma^8, \quad (9)$$

where $\beta_{e^- \rightarrow \nu_i}$ is a dimensionless constant of order unity. [Equation (9) comes from the fact that $I_{e^- \rightarrow \nu_i}$ is approximately equal to the interaction rate ($\Gamma = n\langle\sigma v\rangle \sim G_F^2 T^5$) times the heat capacity ($\sim T^3$).] Three distinct processes contribute to the net value of $I_{e^- \rightarrow \nu_i}$: $e^- e^+$ annihilation into $\nu_i \bar{\nu}_i$ pairs (along with the inverse annihilation), e^- scattering off of ν_i , and e^- scattering off of $\bar{\nu}_i$. The values for $\beta_{e^- \rightarrow \nu_i}$ can be derived from the high-temperature limit of the results of Rana and Mitra [6] and are listed in Table 1, where we have included both the total $\beta_{e^- \rightarrow \nu_i}$ used in equation (9), as well as the contribution to $\beta_{e^- \rightarrow \nu_i}$ from the individual reactions. It is obvious from the numbers in Table 1 that $e^- e^+$ annihilation is the dominant energy transfer mechanism.

In calculating the relative temperatures of ν_e , ν_μ , and ν_τ we must also consider the rate of energy transfer from the ν_e into the other two neutrino types, a process neglected in reference [6]. Again, we can model these rates in terms of an energy loss term w_{ν_i} , $i = \mu, \tau$, where

$$w_{\nu_i} = 2I_{\nu_e \rightarrow \nu_i}(T_{\nu_e} - T_{\nu_i}) \quad (10)$$

and $I_{\nu_e \rightarrow \nu_i}$ is a function of T_{ν_e} which can be written in the form

$$I_{\nu_e \rightarrow \nu_i} = \beta_{\nu_e \rightarrow \nu_i} G_F^2 T_{\nu_e}^8 \quad (11)$$

Again, there are three processes which contribute to the net energy transfer from ν_e into ν_i : $\nu_e \bar{\nu}_e \leftrightarrow \nu_i \bar{\nu}_i$, $\nu_e \nu_i \rightarrow \nu_e \nu_i$, and $\nu_e \bar{\nu}_i \rightarrow \nu_e \bar{\nu}_i$. Using the standard matrix elements (see, e.g., reference [8]), we have calculated the differential cross sections for these three processes. These cross sections can then be substituted into the integral expressions for I corresponding to those given in reference [6] to derive the values for $\beta_{\nu_e \rightarrow \nu_i}$ in equation (11). These values are given in Table 1. Again, we see that $\nu_e \bar{\nu}_e$

Table 1: The coefficients for the energy transfer rates in equations (9) and (11), where ν_i denotes a μ or τ neutrino. We have included the breakdown of $\beta_{e^- \rightarrow \nu_i}$ into specific reactions as well as the total $\beta_{e^- \rightarrow \nu_i}$ used in equations (9) and (11).

Process	β
$e^- \nu_e \rightarrow e^- \nu_e$	0.14
$e^- \bar{\nu}_e \rightarrow e^- \bar{\nu}_e$	0.04
$e^- e^+ \leftrightarrow \nu_e \bar{\nu}_e$	0.99
$e^- \rightarrow \nu_e$	1.17
$e^- \nu_i \rightarrow e^- \nu_i$	0.02
$e^- \bar{\nu}_i \rightarrow e^- \bar{\nu}_i$	0.02
$e^- e^+ \leftrightarrow \nu_i \bar{\nu}_i$	0.21
$e^- \rightarrow \nu_i$	0.25
$\nu_e \nu_i \rightarrow \nu_e \nu_i$	0.07
$\nu_e \bar{\nu}_i \rightarrow \nu_e \bar{\nu}_i$	0.01
$\nu_e \bar{\nu}_e \leftrightarrow \nu_i \bar{\nu}_i$	0.42
$\nu_e \rightarrow \nu_i$	0.50

annihilation is the dominant energy transfer mechanism. With this additional energy loss term, equation (7) becomes:

$$\dot{\rho}_{\nu_e} = u_{\nu_e} - \sum_{i=\mu,\tau} w_{\nu_i} - 4H\rho_{\nu_e}, \quad (12)$$

$$\dot{\rho}_{\nu_i} = u_{\nu_i} + w_{\nu_i} - 4H\rho_{\nu_i} \quad (i = \mu, \tau). \quad (13)$$

In deriving these equations, we have made two major approximations: first, we have neglected the distortion in the energy density spectrum of the neutrinos, treating the spectrum as a blackbody and considering only the change in the total energy density. This is relatively unimportant for the μ and τ neutrinos, since their only effect on nucleosynthesis is via their contribution to the total energy density, but it could be important for the electron neutrinos, since the weak rates are highly energy dependent. Second, equations (8) and (10) are strictly valid only in the limit of $(T_\gamma - T_{\nu_i})/T_\gamma \ll 1$ and $(T_{\nu_e} - T_{\nu_i})/T_{\nu_i} \ll 1$, respectively; we have approximated the energy transfer rate by extrapolating these equations into a regime where the temperature differences are large. For instance, with $\tau = 1$ sec, and for expansion times $t < 1$ sec (the relevant time range for the freeze-out of the $n \leftrightarrow p$ reactions), we find that $(T_\gamma - T_{\nu_{\mu,\tau}})/T_\gamma < 0.5$, $(T_\gamma - T_{\nu_e})/T_\gamma < 0.35$ and $(T_{\nu_e} - T_{\nu_{\mu,\tau}})/T_{\nu_e} < 0.25$. The temperature differences are smaller for shorter lifetimes. For $\tau < 0.1$ sec we have

$(T_\gamma - T_{\nu_{\mu,\tau}})/T_\gamma < 0.2$, $(T_\gamma - T_{\nu_e})/T_\gamma < 0.1$ and $(T_{\nu_e} - T_{\nu_{\mu,\tau}})/T_{\nu_e} < 0.1$. So we expect our approximation to be quite accurate for $\tau < 0.1$ sec, but less accurate for larger lifetimes.

We have integrated our temperature evolution equations for a decaying particle with $rm_X = 10^4$ MeV. Figure 1 shows the evolution of the neutrino temperatures as a function of time for the case of $\tau = 0.1$ sec, including the effect of e^-e^+ annihilation. The temperatures are displayed relative to the photon temperature in terms of the fraction T_{ν_i}/T_γ . We see that for $t < \tau$, both T_{ν_e}/T_γ and $T_{\nu_{\mu,\tau}}/T_\gamma$ decrease as the particle decays, but the electron neutrinos are held closer to the photon temperature than the μ and τ neutrinos, as in reference [3]. The decrease in T_{ν_i}/T_γ which occurs at $t > 10$ sec is the standard effect from the annihilation of the e^-e^+ pairs and has nothing to do with the particle decay. An interesting “rebound” effect occurs between 10^{-1} and 1 sec. The neutrino temperatures actually increase briefly relative the photon temperature before the e^-e^+ entropy dump sets in. We believe that this is a real effect, not an artifact of the approximations we have used. It arises when the rate of energy transfer to the photons (from the decaying particle) is greater than the rate of energy transfer from the photons to the neutrinos. Consider, for example, the (unphysical) limit where the energy of the decaying particles is transferred instantaneously into the photons. After a sharp increase in the photon temperature, the ratio T_ν/T_γ would increase as photon energy is transferred to the neutrinos.

Figure 2 shows the final temperature ratios, multiplied by $(11/4)^{1/3}$ to factor out the effect of e^-e^+ annihilation, as a function of X particle lifetime. Also shown for comparison are the results of the sudden decoupling approximation used in reference [3]. This plot demonstrates significant differences between these two treatments. The sudden decoupling approximation produces a large difference between the electron and other neutrino temperatures in the region $\tau = 0.1$ sec. In our more exact treatment, the differential heating effect is reduced, but it persists over a larger range in lifetime, for τ as large as 1 sec. This is what one might have expected, since the more exact treatment allows for energy transfer from photons to neutrinos at late times, when the sudden decoupling approximation assumes that the neutrinos are already decoupled.

III. EFFECTS ON ^4HE PRODUCTION AND CONCLUSIONS

We have applied these equations to the primordial nucleosynthesis computer code of Wagoner [9], as modified by Kawano [10]. For a decaying particle with energy

density $rm_X = 10^4$ MeV and lifetime τ , we have calculated the change in the primordial helium mass fraction ΔY as a function of τ for three values of the baryon to photon ratio: $\eta = 10^{-10}, 10^{-9.5}$, and 10^{-9} . These results are displayed in Fig. 3. We find that ΔY is nearly independent of η and can be as large as -0.012 . We see that $\Delta Y < 0$ for $\tau < 1.5$ sec; for larger lifetimes the decaying X particle, rather than its thermalized decay products, dominates the expansion during the freeze-out of the $n \leftrightarrow p$ reactions, resulting in a larger expansion rate and a net increase in ${}^4\text{He}$.

For comparison, in Figure 4 we include the ΔY value for $\eta = 10^{-9.5}$ using the sudden decoupling approximation from reference [3]. We find that our more exact treatment produces a smaller decrease in the primordial helium, but that the effect occurs over a larger range in particle lifetime; we find that $\Delta Y < -0.005$ for $\tau = 0.05\text{--}1.2$ sec. This is not surprising, since our more exact treatment produces a smaller differential heating effect spread out over a larger range in τ . Part of the difference, however, is that the actual total interaction rates used to compute decoupling in references [3]-[4] are different from those used here. So in Figure 4 we also give the helium abundances corresponding to sudden decoupling when we change the total interaction rates in equation (5) to give the same freeze-out temperatures as those derived in ref. [6] ($\alpha_{\nu_e} = 1.5$, $\alpha_{\nu_{\mu,\tau}} = 0.33$). The curve is shifted over to correspond more closely to our more exact treatment, but it retains a much sharper and deeper minimum.

Of the ν_τ decay modes considered by Dodelson et al. [2], our present work most closely resembles ν_τ decay into sterile plus electromagnetic decay products. No reduction in ${}^4\text{He}$ is evident for this case in reference [2], but this is to be expected, because the ν_τ energy density is too low to lead to significant differential neutrino heating. Dodelson et al. [2] do see a significant reduction in ${}^4\text{He}$ for decay modes which include a ν_e in the final state, but this reduction occurs for entirely different reasons: the additional ν_e decay products increase the $n \leftrightarrow p$ weak rates, keeping them in thermal equilibrium longer.

Our results indicate that the decrease in ${}^4\text{He}$ production due to differential neutrino heating from a decaying particle, first discussed in reference [3], holds up under a more exact treatment. The effect is not as large as had been previously estimated, but it occurs over a longer range in particle lifetime. The change in Y can be as large as -0.01 for τ in the range $0.1 - 0.7$ sec, and as large as -0.005 for $\tau = 0.05 - 1.2$ sec. A decrease on the order of 0.01 can resolve current problems with

standard primordial nucleosynthesis [5], although it is perhaps not the most plausible mechanism for resolving these problems. A more detailed treatment of the spectral distortion using the full machinery of reference [2] seems justified, although we do not expect our results to be significantly altered in such a treatment. Even the treatment in reference [2] would need to be modified to treat this problem correctly, since that treatment also assumes small temperature differences.

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References

- [1] M. Kawasaki, P. Kernan, H.-S. Kang, R.J. Scherrer, G. Steigman, and T.P. Walker, Nucl. Phys. B **419**, 105 (1994).
- [2] S. Dodelson, G. Gyuk, and M.S. Turner, Phys. Rev. D **49**, 5068 (1994).
- [3] R.J. Scherrer and M.S. Turner, Ap.J., **331**, 19 (1988).
- [4] R.J. Scherrer, J. Cline, S. Raby, & D. Seckel, Phys. Rev. D, **44**, 3760 (1991).
- [5] C. Copi, D.N. Schramm, and M.S. Turner, Science, **267**, 192 (1995); N. Hata, R.J. Scherrer, D. Thomas, T.P. Walker, S. Bludman, and P. Langacker, Phys. Rev. Lett., submitted; C. Copi, D.N. Schramm, and M.S. Turner, Phys. Rev. Lett., submitted.
- [6] N.C. Rana and B. Mitra, Phys. Rev. D, **44**, 393 (1991).
- [7] A.M. Herrera and S. Hacyan, Phys. Fluids **28**, 3253 (1985); A.M. Herrera and S. Hacyan, Astrophys. J. **336**, 539 (1989).
- [8] S. Dodelson and M.S. Turner, Phys. Rev. D, **46**, 3372 (1992).
- [9] R.V. Wagoner, W.A. Fowler, and F. Hoyle, Astrophys. J. **148**, 3 (1967); R.V. Wagoner, Astrophys. J. Suppl. Ser. **162**, 247 (1969); Astrophys. J. **179**, 343 (1973).
- [10] L. Kawano, FERMILAB-Pub-92/04-A (1992).

FIGURE CAPTIONS

FIG. 1. The time evolution of T_{ν_i}/T_γ for electron neutrinos (upper curve) and μ and τ neutrinos (lower curve) with a decaying particle with lifetime $\tau = 0.1$ sec and energy density parameter $rm_X = 10^4$ MeV.

FIG. 2. The final ratios $(11/4)^{1/3}(T_{\nu_i}/T_\gamma)$ as a function of particle lifetime τ for a decaying particle with a density parameter $rm_X = 10^4$ MeV (solid curves). For comparison, the corresponding ratios are also shown for the sudden decoupling approximation [3] (dashed curves).

FIG. 3. The change in the primordial ^4He abundance, ΔY , as a function of decaying particle lifetime τ for a decaying particle with a density parameter $rm_X = 10^4$ MeV and three different baryon-to-photon ratios: $\eta = 10^{-10}$ (dashed curve), $\eta = 10^{-9.5}$ (solid curve) and $\eta = 10^{-9}$ (dotted curve).

FIG. 4. The change in the primordial ^4He abundance, ΔY , for $\eta = 10^{-9.5}$, as a function of decaying particle lifetime τ for a decaying particle with a density parameter $rm_X = 10^4$ MeV (solid curve) compared with the results of the sudden decoupling approximation [3] (dashed curve) and the sudden decoupling approximation with the interaction rates updated to correspond to the ones used here (dotted curve).

Figure 1

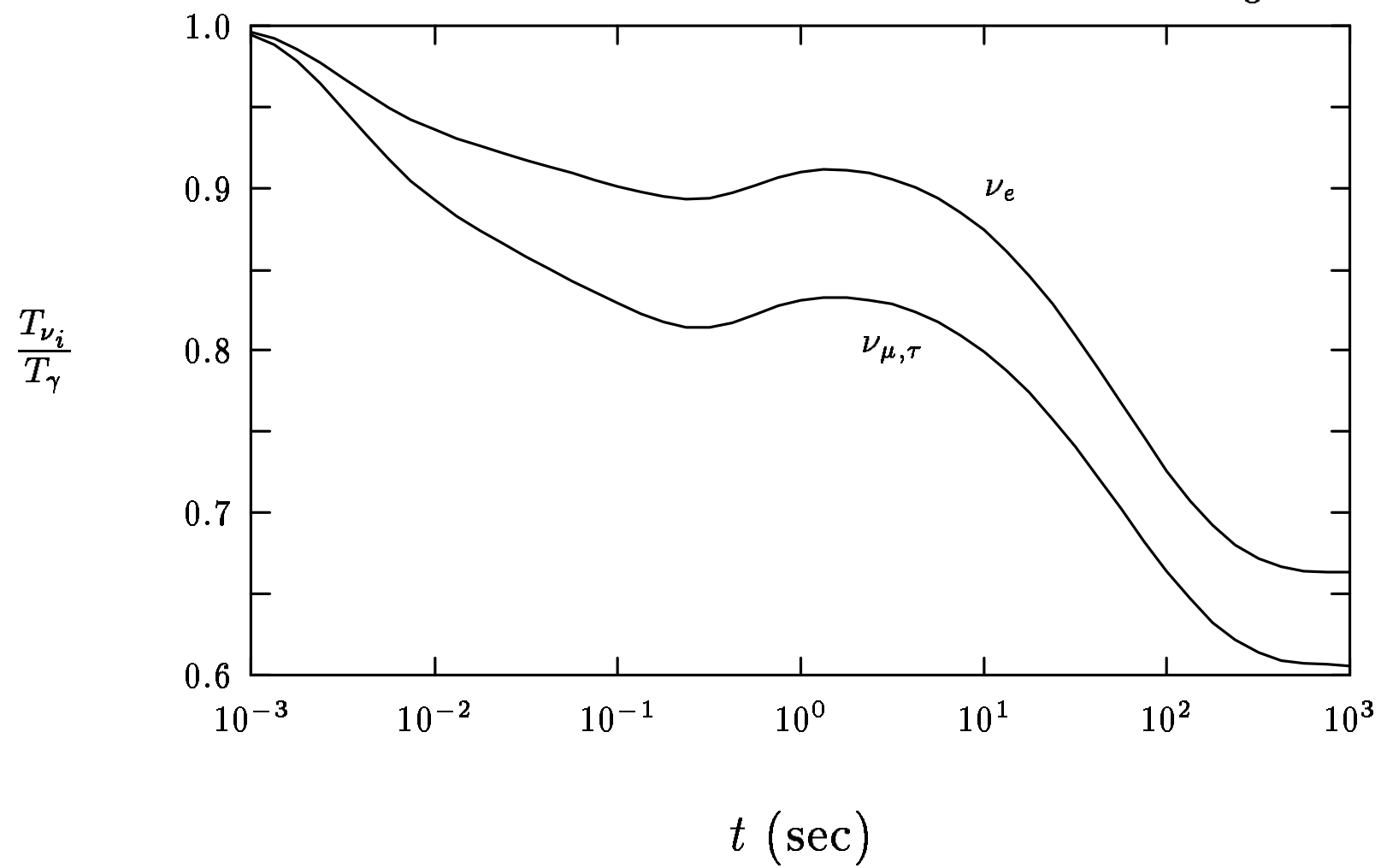


Figure 2

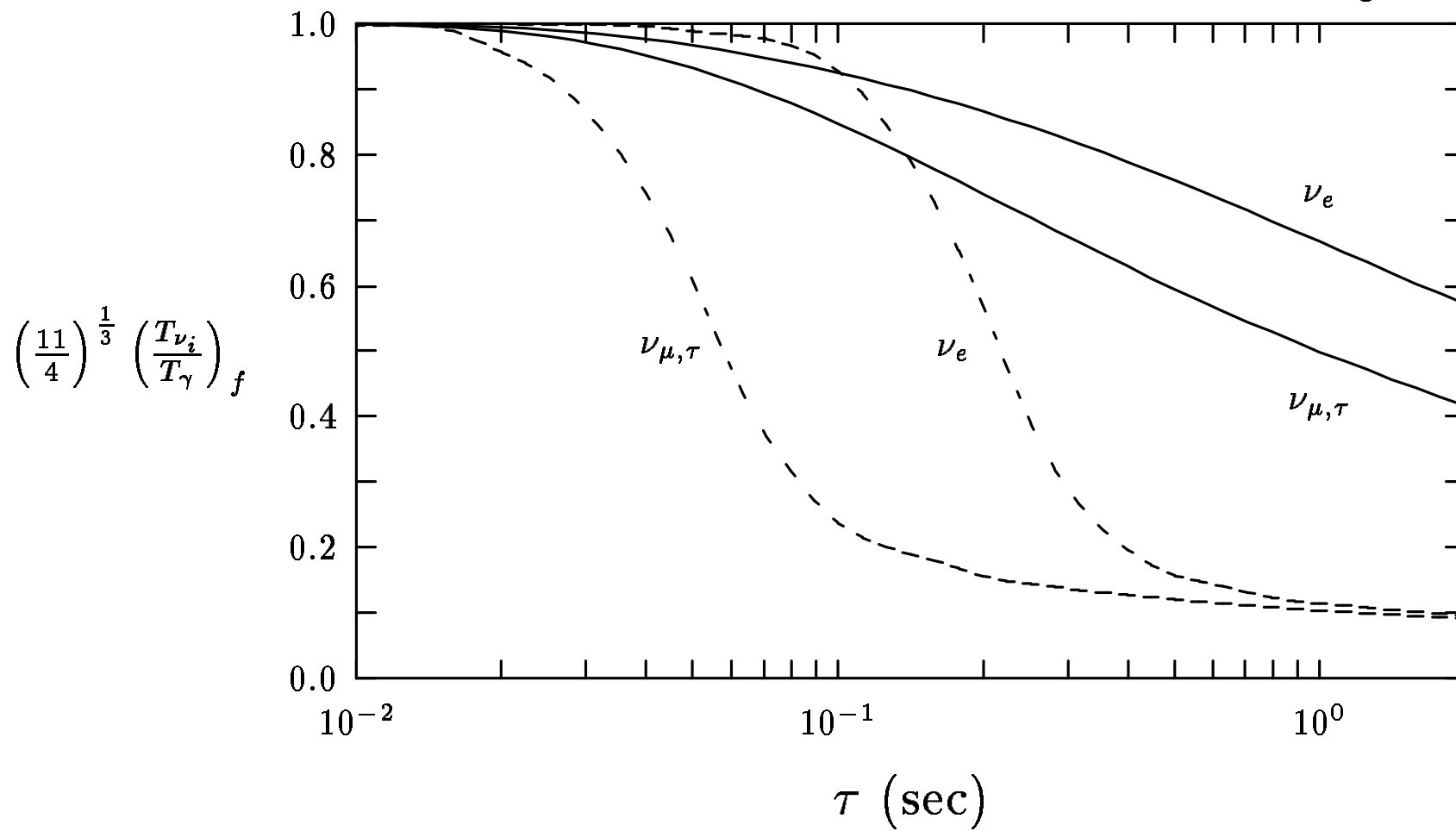


Figure 3

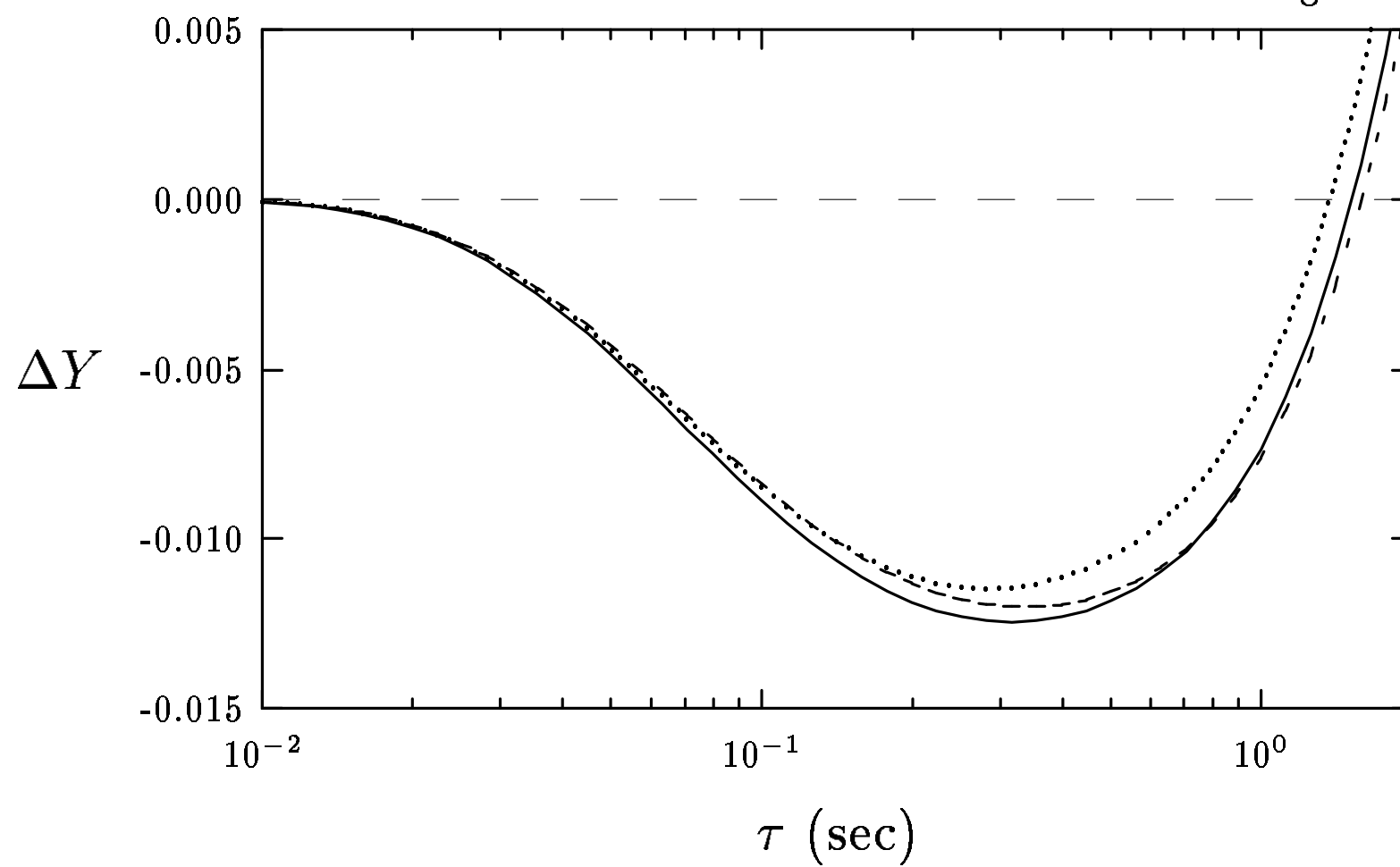


Figure 4

